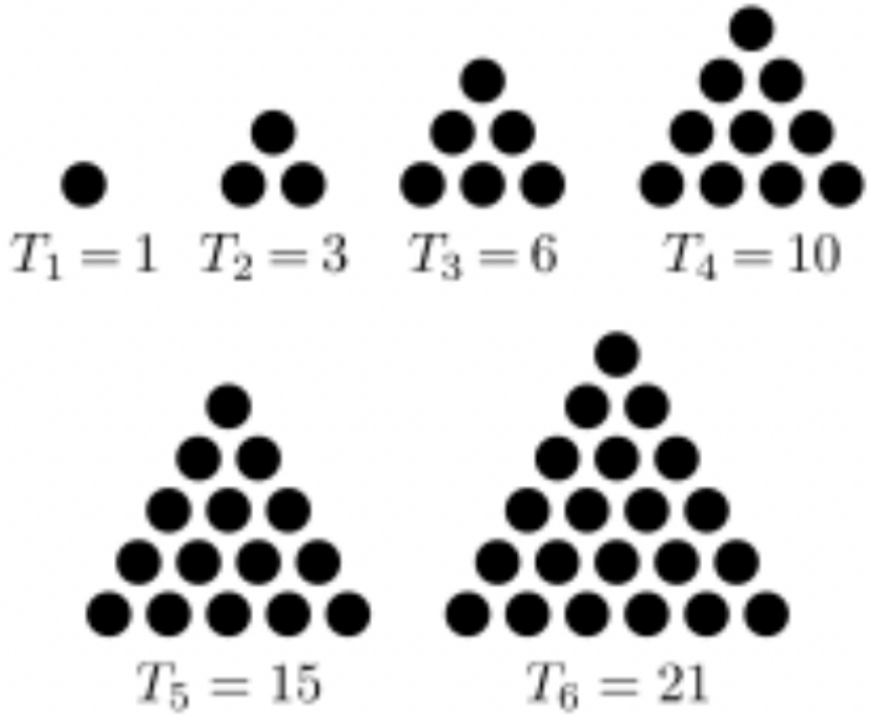
Math Circle 10/19/2022

Average of triangular numbers

The n^{th} triangular number is $T(n) = \frac{n \times (n+1)}{2}$.



The first six triangular numbers

For which n is the average, $\frac{1}{n}\sum_{k=1}^{n} T(k)$, of the first n triangular numbers $T(1), T(2), \ldots, T(n)$ an integer?

Sum of squares of consecutive integers

We know that $3^2 + 4^2 = 5^2$.

What is probably not so well known is that $20^2 + 21^2 = 29^2$.

For which other positive integers n is it true that $n^2 + (n+1)^2$ is the square of a positive integer?

Is there some way to recursively determine such positive integers n?